HY-330

fall semester 2024

Introduction to telecommunication systems theory

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Fourier Transform*

*since 1822

- Time domain - Frequency domain
- Discrete Fourier Transform
- Fast Fourier Transform
- Practical Applications
- Windowing
- Examples

Fourier Transform

- Function of time -> decompose to frequencies
- Fourier Transform
  - frequency domain representation
  - time domain to frequency domain operation
- Linear operations have both time & frequency domain counterparts

Fourier Transform (discrete)

- Fourier Transform

$$X_k = \sum_{n=0}^{N-1} x_n \cdot (\cos(-2\pi k \frac{n}{N}) + j\sin(-2\pi k \frac{n}{N})), k \in \mathbb{Z}$$

time sample

- Inverse Fourier Transform

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k \cdot (\cos(2\pi k \frac{n}{N}) + j\sin(2\pi k \frac{n}{N})), n \in \mathbb{Z}$$

frequency component

Fast Fourier Transform

- DFT complexity $O(n^2)$
- FFT complexity $O(n \log n)$

Time - Frequency Domain

- Time Domain

$$T = N \cdot \Delta t$$

$$\Delta f = \frac{1}{T}$$

- Frequency Domain

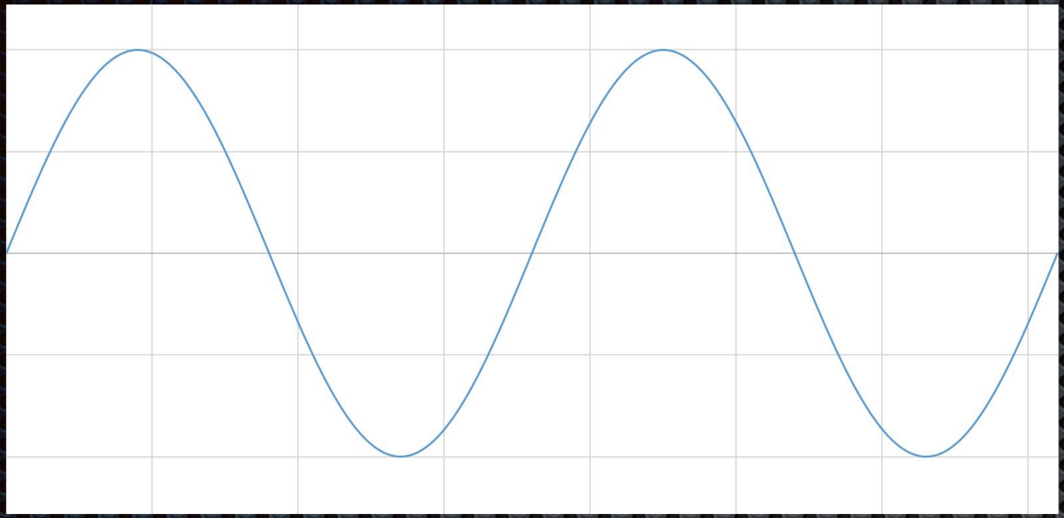
$$F_{max} = \frac{N}{2} \cdot \Delta f$$

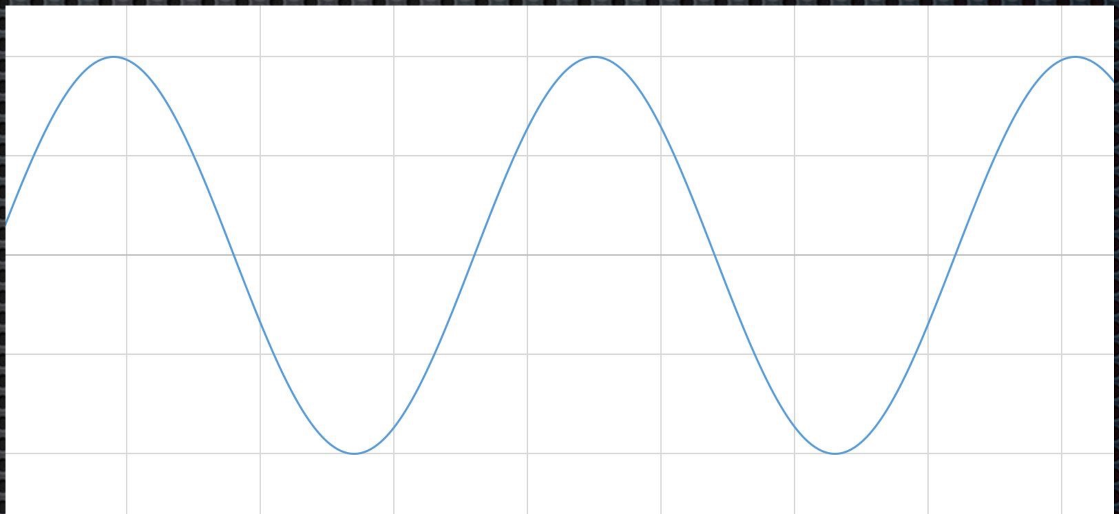
$$\Delta t = \frac{1}{2 \cdot F_{max}}$$

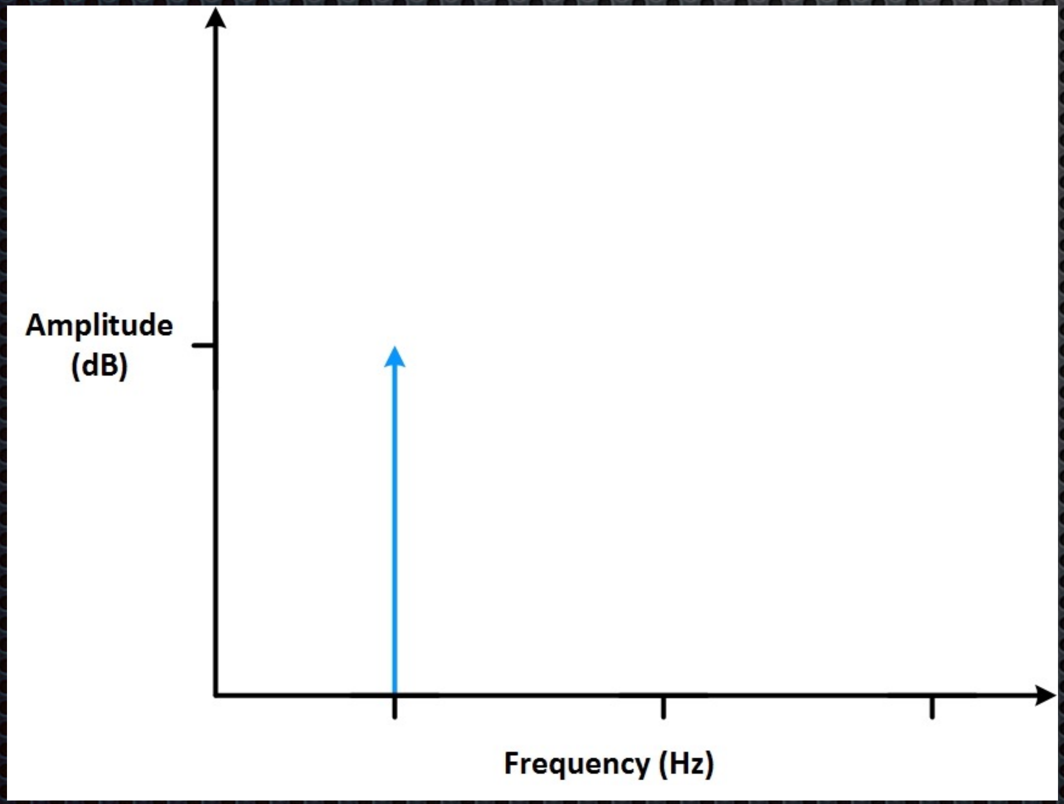
Problem

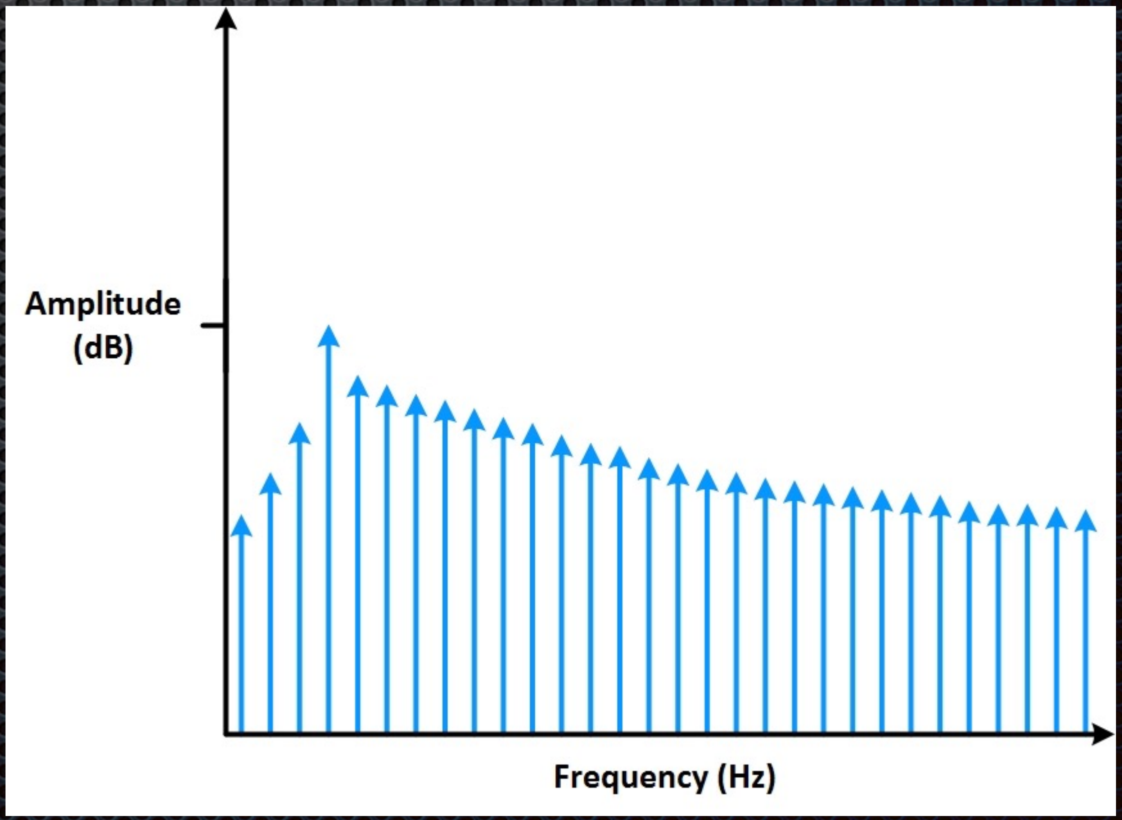
- FFT assumes a finite data set, a continuous spectrum of integer periods of a periodic signal
- What if the number of samples are not providing integer number of periods?
  - Discontinuities...

Problem





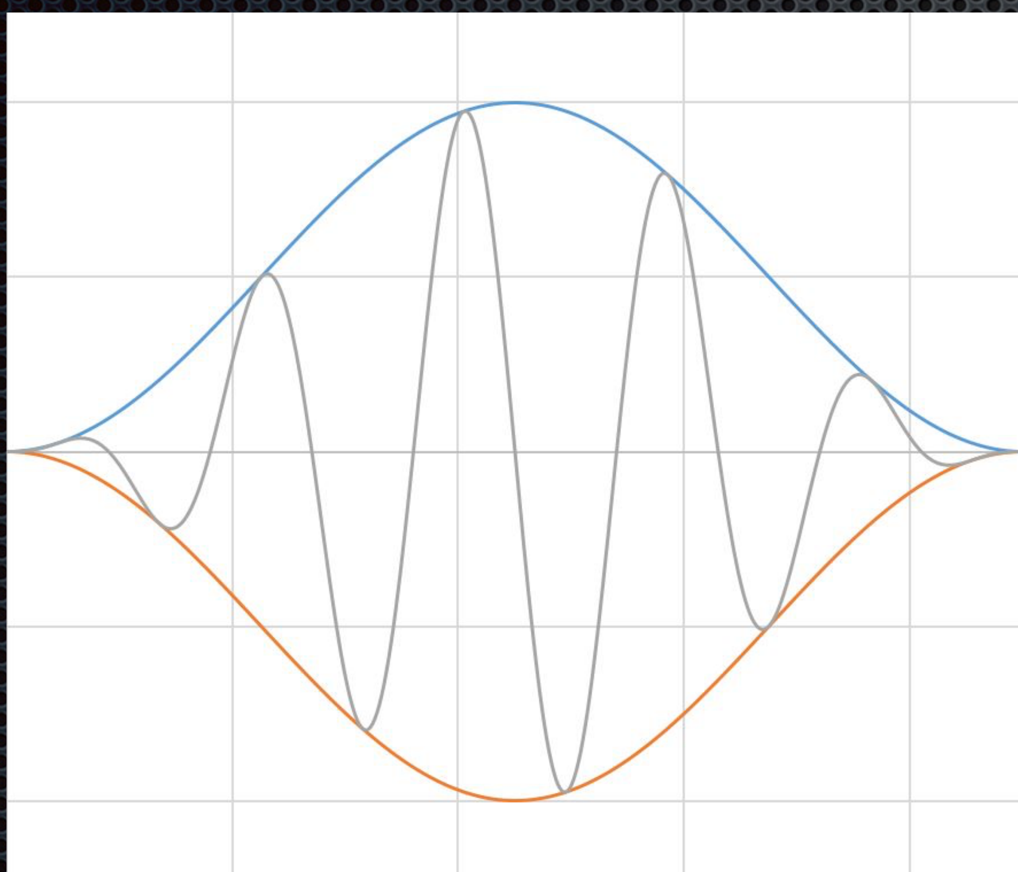


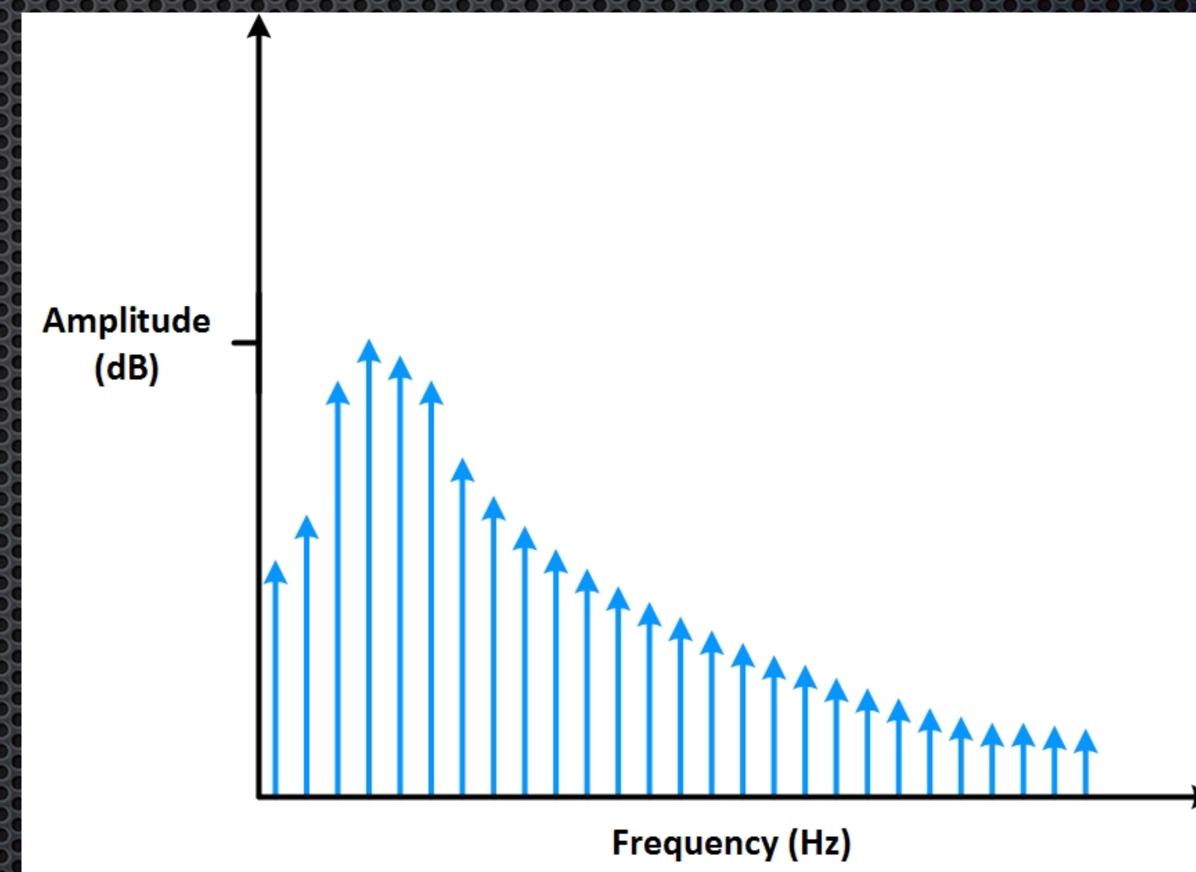


Solution

- Try to minimize the effect of the excess samples
- Suppress the extra samples that are not providing integer number of periods...
- Windowing technique

Solution

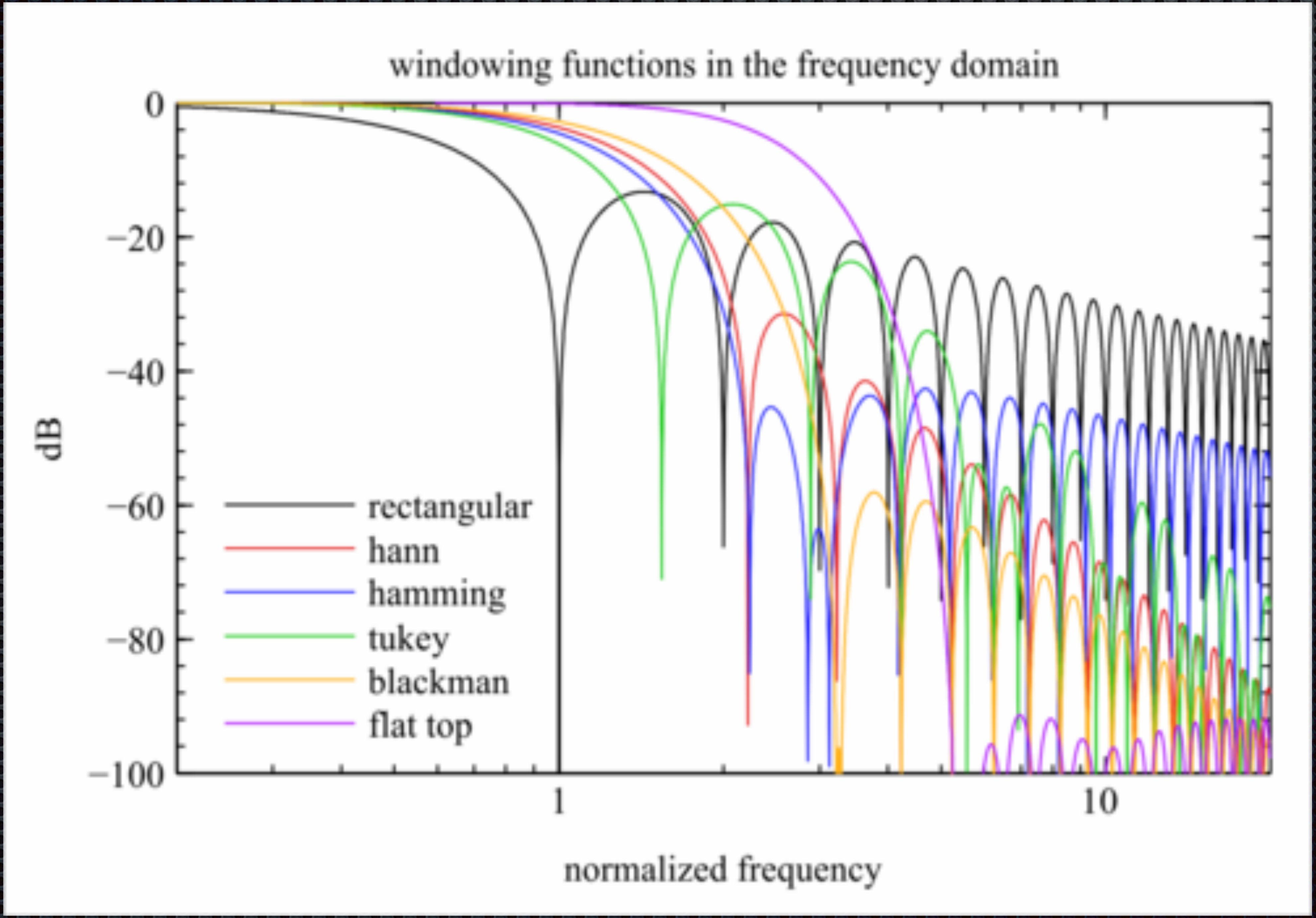




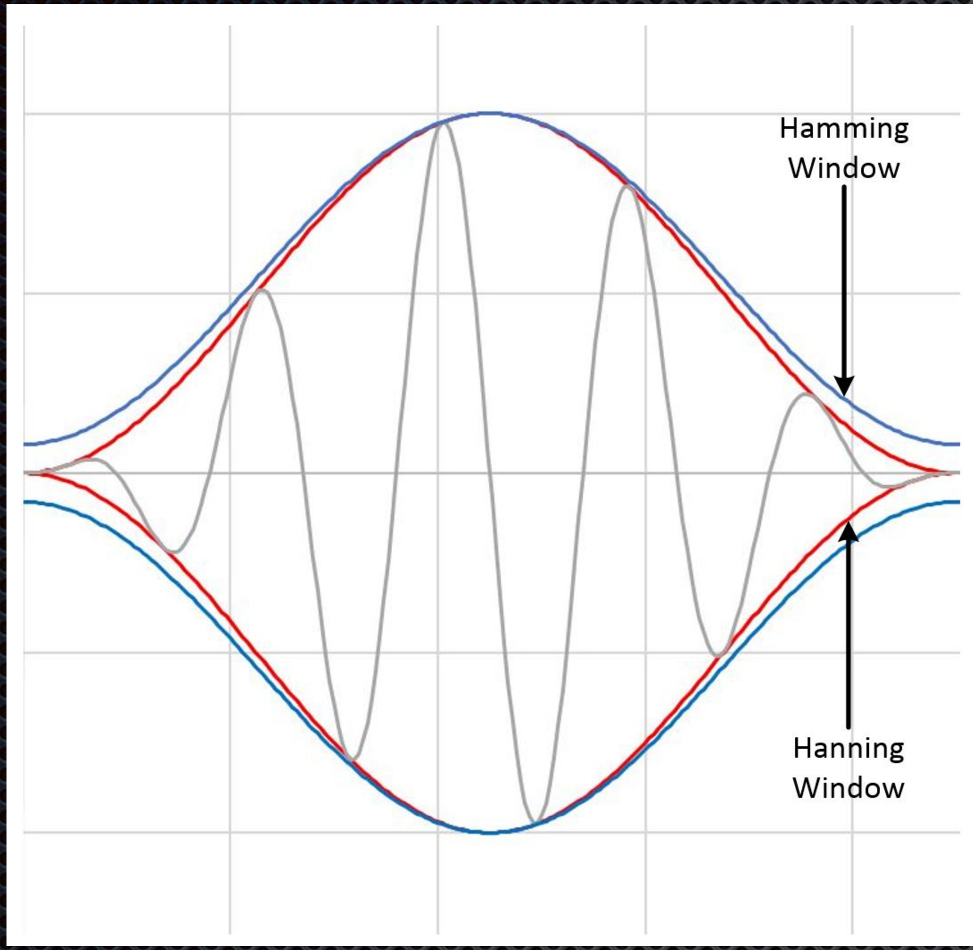
Windowing Functions

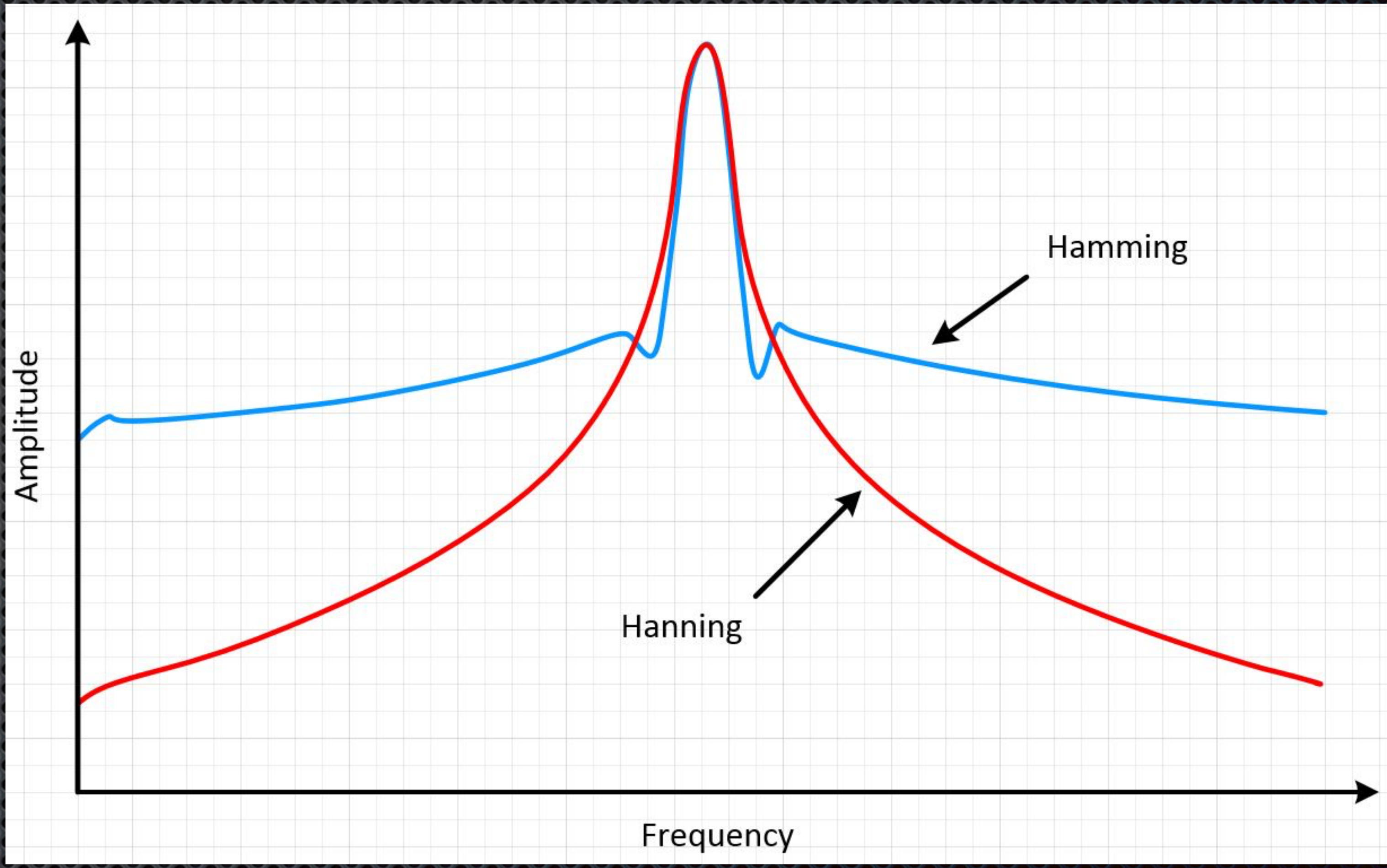
- Rectangular
- Triangular
- Gaussian
- Exponential/Poisson
- Tukey
- Kaiser
- Hann (Hanning)
- Hamming
- Blackman
- Blackman-Harris
- Flat Top
- Planck–Bessel
- Kaiser-Bessel
- Lanczos/sinc

Windowing Functions

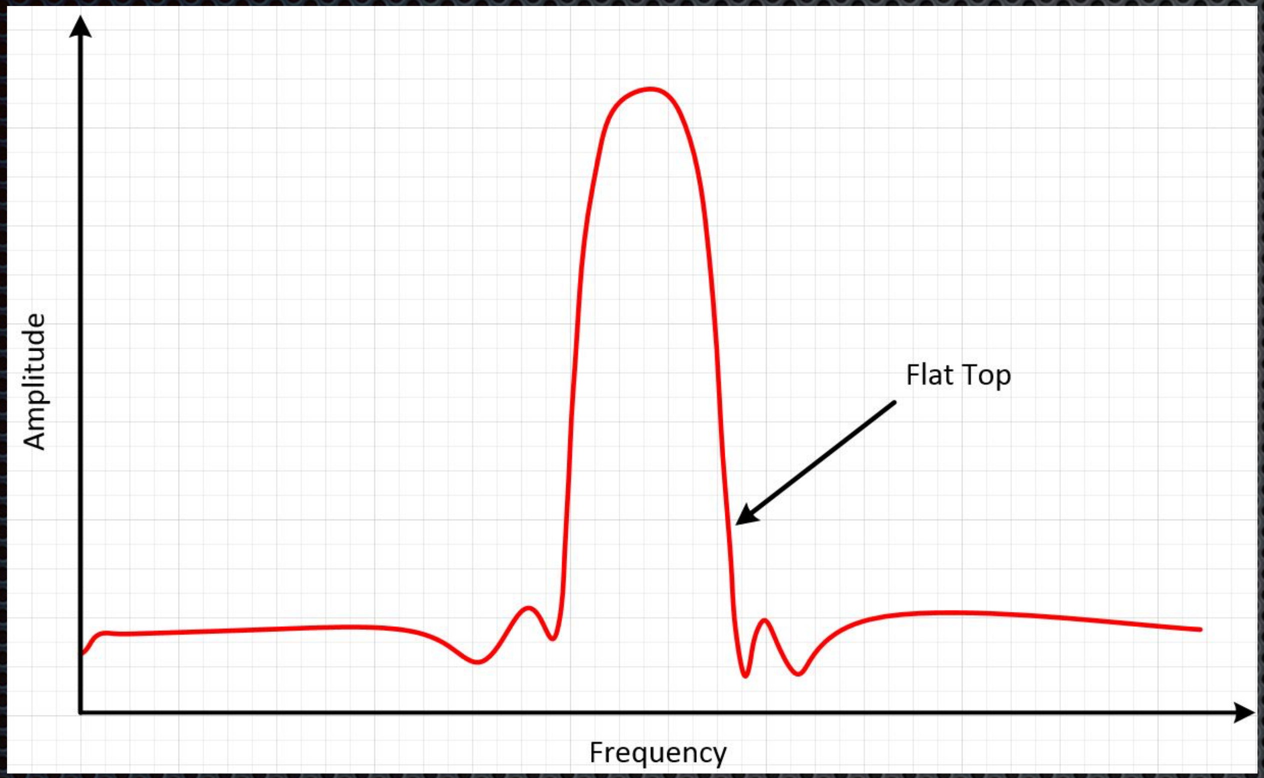


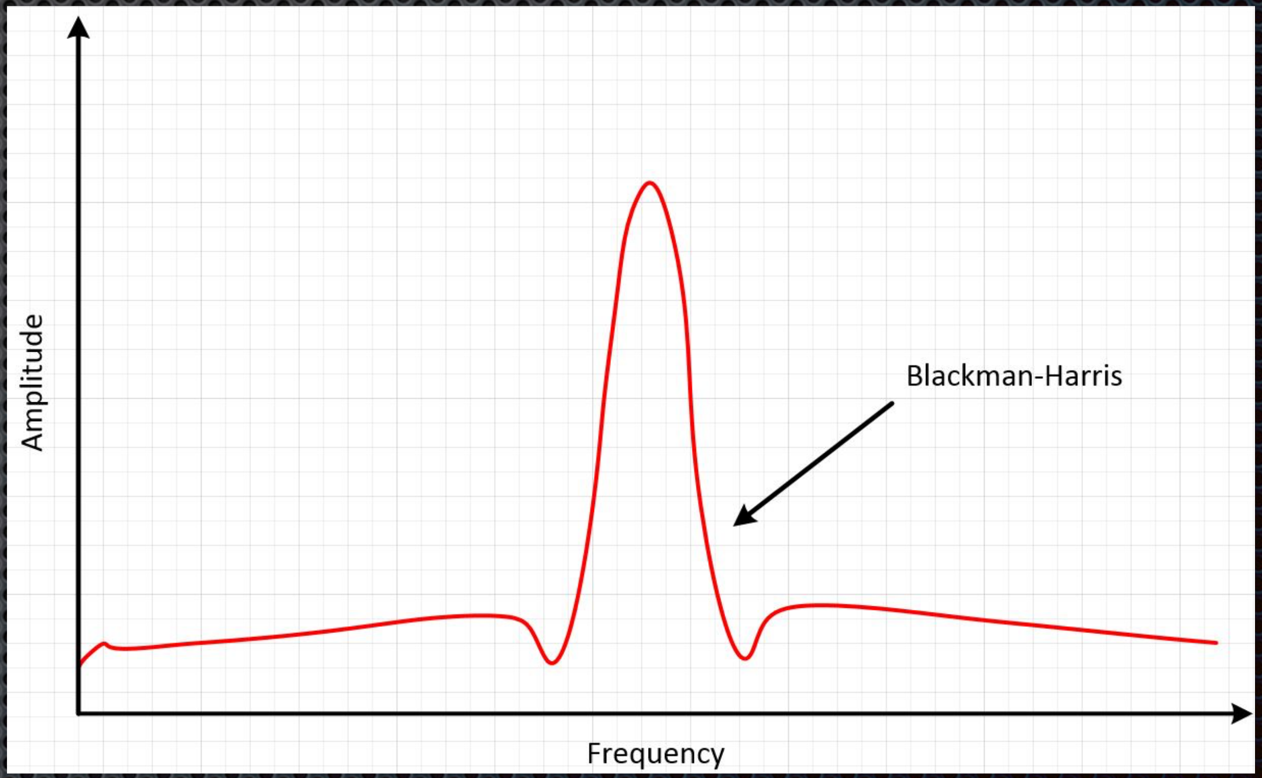
Windowing Functions





Windowing Functions

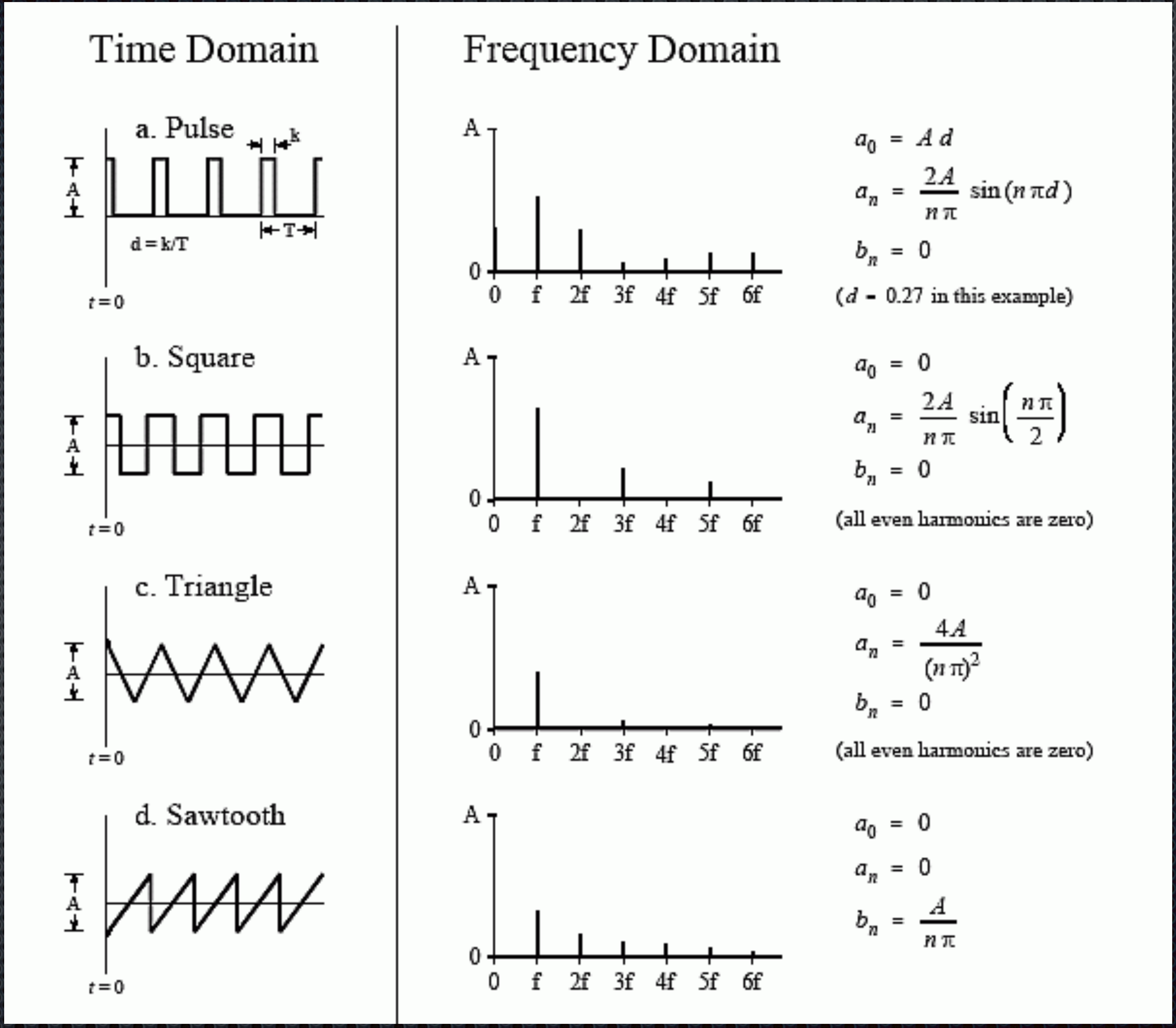




Windowing Functions

Signal Content	Window
Sine wave or combination of sine waves	Hann
Sine wave (amplitude accuracy is important)	Flat Top
Narrowband random signal (vibration data)	Hann
Broadband random (white noise)	Uniform
Closely spaced sine waves	Uniform, Hamming
Excitation signals (hammer blow)	Force
Response signals	Exponential
Unknown content	Hann
Sine wave or combination of sine waves	Hann
Sine wave (amplitude accuracy is important)	Flat Top
Narrowband random signal (vibration data)	Hann
Broadband random (white noise)	Uniform
Two tones with frequencies close but amplitudes very different	Kaiser-Bessel
Two tones with frequencies close and almost equal amplitudes	Uniform
Accurate single tone amplitude measurements	Flat Top

Non-sinusoidal waves



Square Wave